An Examination of the Gamma Match

BY D. J. HEALEY, III,[*] W3PG, ex-W3HEC

A working analysis of the gamma-match problem that gives useful practical results. New light on the question of when it will and when it won't give a perfect match to the coaxial transmission line.

THE gamma match has been widely used for matching coaxial cable to all-metal parasitic beams for the past nineteen years.[1] Recently, a three-element 14-MHz. Yagi, having a boom length of 0.4 wavelength, was constructed at W3PG and mounted atop a 75-foot Union Metal unguyed antenna pole. A driven-element diameter of 1½ inches, a gamma-rod diameter of ⅝ inch, and 5-inch center-to-center spacing between the gamma rod and driven element were employed. A capacitance patterned after the W2VS design[2] was also provided. The maximum distance provided between the short and the antenna center was 44 inches.

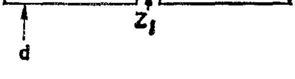
When attempts were made to adjust the coaxial capacitor for a match to 50 ohms (using a resistance bridge connected to the transmission line approximately four feet from the matching section) a complete null could not be achieved. Variation of the capacitor resulted in best match at maximum capacitance, approximately 160 pf. Measurement of standing-wave ratio at the transmitter end of the coaxial cable showed a minimum s.w.r. of 1.5 to 1 at 14.0 MHz. and an increase to 3 to 1 at 14.35 MHz.

The author had used the gamma match on a number of beams in the past with much better results, but with somewhat different dimensions. In reviewing various articles on antennas that have appeared in *QST* and *CQ*, there appears to be quite a variation in the diameter ratios and spacing between gamma rod and driven element that have been employed. Nowhere in the literature, however, was I able to find a discussion of the gamma match that would indicate the limitations of the device. Since working at the top of the pole is rather difficult, I decided that it might be a good idea to have a better understanding of the matching section before making changes. What follows constitutes my interpretation of the operation of the gamma match.

Input Impedance of the Antenna

The input impedance at the center of a half-wave antenna which has infinite length-to-diameter ratio and zero conductor resistance is 73 + j42 ohms. This is the radiation impedance. A practical antenna has conductor resistance and a finite length-to-diameter ratio. Typically, the input impedance at the center of a half-wave dipole made of aluminum tubing is about 68 + j35 ohms and the conductor resistance is negligible. To make the dipole resonant, it is necessary to shorten it. Fig. 1 shows the length that is required.[3] It is not *necessary* that the dipole be resonant — in fact, when using the gamma match this is in many cases rather undesirable!

When the dipole is the driven element of a Yagi parasitic array, the impedance that appears at the center of the driven element will usually be quite different from that of the isolated dipole. The reason for this is that the Yagi antenna is equivalent to a number of resonant circuits tuned to different frequencies and coupled together. The self-impedance of the parasitic elements and mutual impedance between parasitic elements and driven element cause resistance and reactance to be coupled into the driven element. Mushiake[4] has made a theoretical analysis for the three-element wide-spaced Yagi in which reflector and driven element are a half wavelength long and reflector-to-driven-element spacing is fixed at a quarter wavelength. His results show maximum power gain occurring when the director-to-driven-element spacing is 0.2 wavelength and the director length is 0.45 wavelength. Changing director spacing and length



FOR RESONANCE,

$$\ell = \frac{0.48\lambda\, \ell/d}{\ell/d + 1}$$

Fig. 1—Simple dipole with formula for resonant length.

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[1] Washburn, "The 'Gamma' Match," *QST*, September, 1949.

[2] Reynolds, "Simple Gamma-Match Construction," *QST*, July, 1957.

[3] Kraus, *Antennas*, p. 276, McGraw-Hill Book Co., Inc., New York, 1950.

[4] Mushiake, "A Theoretical Analysis of the Multi-Element End-Fire Array with Particular Reference to the Yagi-Uda Antenna," *IRE Transactions on Antennas & Propagation*, July, 1956, pp. 441–444.

List of Symbols

b_p — Susceptance of X_p.
d_1 — Diameter of gamma rod.
d_2 — Diameter of driven element.
e — Length of driven element.
S — Center-to-center spacing of element and gamma rod.
X_p — Parallel reactance of gamma considered as shorted line.
X_Γ — Reactance needed for compensating gamma input reactance.
Y'_2 — Admittance of Z_2'.
Y_Γ — Input admittance of gamma.
Z_o — Characteristic impedance of gamma section.
Z_1 — Impedance at center of driven element.
Z_2 — Impedance of driven element at tap point.
Z_2' — Z_2 transformed to input of gamma.
Z_Γ — Input impedance of gamma.
θ — Angular distance from center to tap on driven element.

from 0.15λ and 0.46λ, respectively, to 0.25λ and 0.44λ reduces the power gain by only 0.3 db. from the maximum.

Mushiake gives computed curves of the input impedance which show that for the maximum-gain condition the impedance at the center of the driven element is 26 + j60 ohms. Thus, as compared with a half-wave dipole, the radiation resistance is reduced and the reactance is increased due to the coupling between the driven element and parasitic elements. As the director length and spacing are varied over the limits, stated above, that affect gain by only 0.3 db., the impedance varies from about 20 + j40 to 38 + j70 ohms.

In typical amateur construction the impedance will probably tend to be on the lower side. The problem then is to couple the transmission line to the driven element (which will have an impedance of 20 to 25 ohms of resistance in series with a reactance which will depend on the driven-element length) in such a way that a low s.w.r. will exist on the transmission line over the operating bandwidth.

From Mushiake's curves it is obvious that the resistance and reactance will change more rapidly with a change in frequency than in the case of an isolated dipole.

The Gamma Match

Fig. 2 shows the basic gamma-matching section. The driven element is tapped at a point off center. By making the tapping conductor parallel to the driven element and closely spaced to it, several electrical characteristics result:

1) The antenna current flowing in the driven element is less than when the gamma rod is not present — i.e., when the element is excited as split dipole. Part of the antenna current flows through the gamma rod. The result is an effective increase in the input impedance. Fig. 3 is a set of curves from the *ARRL Antenna Book* which gives the impedance step-up for folded-dipole antennas. This same set of curves can be used to determine the impedance step-up of the gamma-fed driven element.

2) Since the driven element is fed off center, the impedance at the tap point is higher than when it is fed as a split element. The increase in impedance is given approximately by the relationship[5]

$$Z_2 = \frac{Z_1}{\cos^2 \theta} \qquad (1)$$

where Z_2 is the impedance at the tap point, Z_1 is the impedance at the center of the element, and θ is the number of electrical degrees (distance in wavelengths multiplied by 360) between the center of the element and the tap point.

3) Since the gamma rod is parallel to the driven element, the tapped impedance point of the antenna is connected to the transmission-line feed position via a two-conductor parallel-conductor transmission line. As a result, if the impedance at the tapping point of the element is Z_2, the radiation impedance at the exciting end of the gamma is modified to:

$$Z_2' = \frac{Z_2 + jZ_o \tan \theta}{1 + \frac{jZ_2}{Z_o} \tan \theta} \qquad (2)$$

where Z_o is the characteristic impedance of the short transmission line formed by the driven element and the gamma rod. The characteristic impedance is

$$Z_o = 276 \log_{10} \frac{2S}{\sqrt{d_1 d_2}} \quad \text{ohms} \qquad (3)$$

where S is is the spacing between centers of driven element and gamma rod, and d_1 and d_2 are the diameters of the driven element and rod.

4) Because the matching arrangement constitutes a short transmission-line section which is shorted at one end insofar as transmission-line currents are concerned (these currents flow in opposite directions in the rod and driven element, and are necessary to excite the antenna current which flows in the *same* direction in the rod and driven element) the transformed radiation im-

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[5] Wrigley, "Impedance Characteristics of Harmonic Antennas," *QST*, February, 1954.

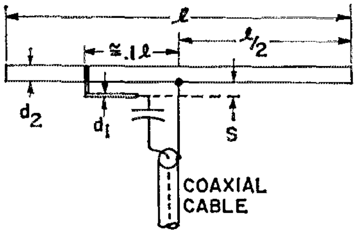

Fig. 2—Gamma match, with significant dimensions.

pedance appearing at the driven end of the gamma match is shunted by a reactance which is

$$X_p = jZ_o \tan \theta \text{ ohms} \qquad (4)$$

Since the gamma rod is only about 0.05 to 0.06 wavelength long, the shunting reactance is inductive. The lumped equivalent circuit for a gamma-matched antenna therefore appears as shown in Fig. 4.

Depending on the reactance associated with Z_2 and on the magnitude of Z_o, the gamma section may or may not be capable of providing an input impedance such that, with a capacitor X_Γ of suitable reactance connected in series with the coaxial cable and gamma rod, the cable will be terminated in 53 + j0 ohms, providing a perfect match for cable such as RG-8/U. (This situation probably led to the use of the omega match.)

A large number of combinations of gamma rod length, spacing S, and driven-element length will provide a match. However, if the length becomes too short, the efficiency and bandwidth of the matching section are poor because of the high current through the shorted transmission-line (gamma) section (small value of X_p). The transmission-line loss is $I_t^2R_t$, where I_t is the circulating transmission-line current and R_t is the loss resistance associated with the gamma-rod and driven element. This power is delivered from the transmitter but is not radiated, so both I_t and the resistance of the matching section should be kept small.

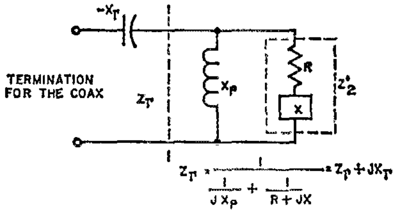

Fig. 4—Equivalent circuit of the gamma. The gamma input impedance is

$$Z_\Gamma = \frac{1}{\frac{1}{jX_p} + \frac{1}{r + jX}}$$

The gamma capacitor provides a reactance $-X_\Gamma$ that cancels the equivalent series reactance of the gamma input impedance, leaving the resistive component as the load for the coaxial transmission line.

The foregoing relationships, as well as the relationships shown in Fig. 4, can be used to determine the operation of the gamma section. However, a more convenient and rapid method of assessing the performance is to examine the effects of varying Z_o, Z_1 and θ on a Smith chart.

This article will not attempt to explain the Smith chart. The reader may find an elementary discussion of the use of the chart in a previous issue of *QST*.[6] Suffice to say, the Smith chart permits equation (2) to be solved for any Z_1, Z_o, and θ very easily, and further makes it very simple to solve the circuit of Fig. 4. In Fig. 4 we know Z_2', having solved equation (2) either by direct computation or by using the Smith chart, and we know X_p, having solved equation (4) or again simply using the Smith chart. The problem is to determine Z_Γ. Fig. 4 shows the equation that must be solved. On a Smith chart this is accomplished by inverting the point Z_2' to obtain $Y_2' = g_2' + jb_2'$. The total admittance, Y_Γ, is then $g_2' + j(b_2' + b_p)$. This point is plotted on the Smith chart, and then inverting this point on the chart yields Z_Γ.

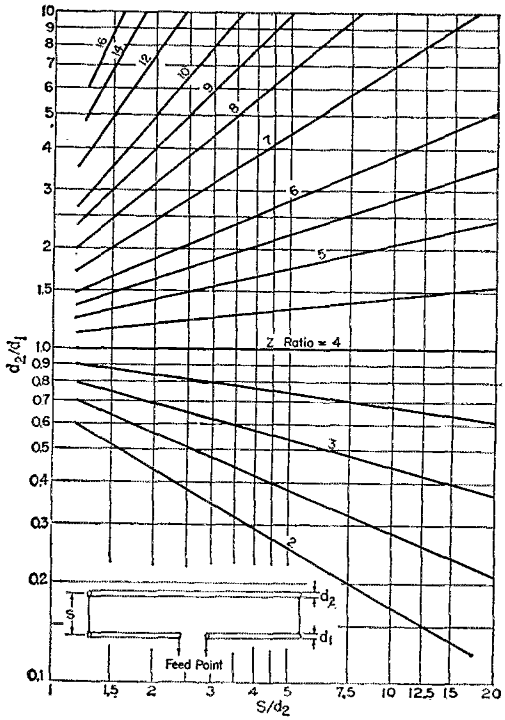

Fig. 3—Impedance step-up ratio as a function of conductor diameter and spacing.

Examples:

One of the points made in discussions of the gamma-matching problem is that the driven element must be resonant.[7, 8] Fig. 5A shows what will happen if the driven element is made resonant. A driven-element diameter of 1½ inches, a gamma rod diameter of ¾ inch and a spacing (S) of 5 inches is considered. This results in Z_o = 258 ohms, and from Fig. 3 an impedance step-up of 5.6 times. Since the driven element is made resonant, the radiation resistance will for the typical three-element Yagi be about 25 ohms. Letting the tap point be about 0.05λ from the center of the element, Z_2 is approximately

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[6] Cholewski, "Some Amateur Applications of the Smith Chart," *QST*, January, 1960.

[7] Nose, "Adjustment of Gamma-Matched Parasitic Beams," *QST*, March, 1958.

[8] Orr, *Beam Antenna Handbook*, Radio Publications, Inc., Wilton, Conn., 1965.

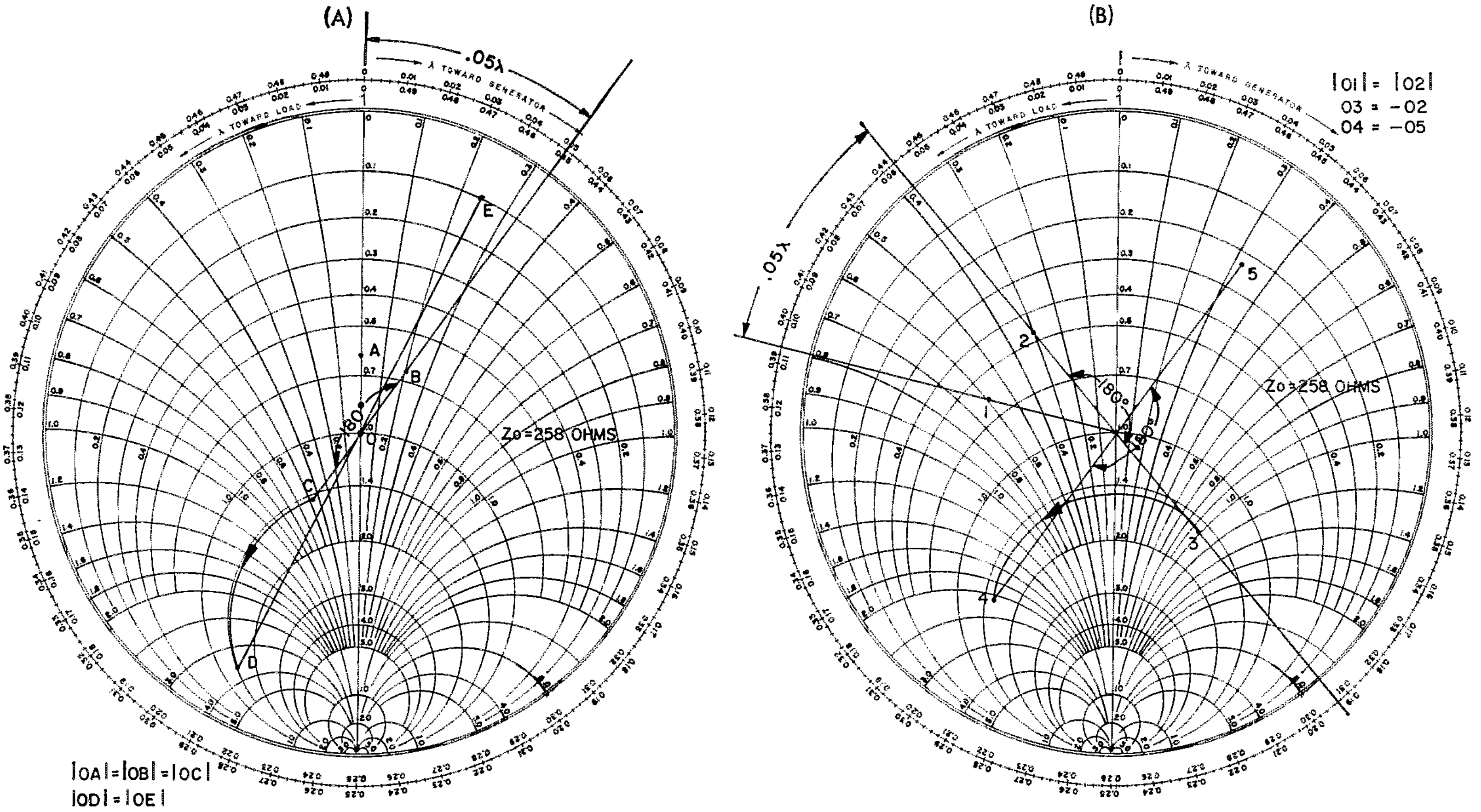


Fig. 5—Smith-chart plot for (A) the case where the driven element length is adjusted to resonance and gamma tap is 0.05λ from center, and (B) for a driven element shortened to provide capacitive reactance at the drive point; tap 0.05λ from center.

$$\frac{25}{\cos^2 (18°)} = 27.7 \text{ ohms.}$$

The stepped-up impedance is 5.6 × 27.7 = 155 ohms. On Fig. 5A, point A is this impedance normalized to the Z_o of the gamma section (i.e., $\frac{155}{258} = 0.6$). Since the length of the gamma section was taken as 0.05λ, the impedance Z_2' is obtained by rotation of point A about the center of the chart by an amount 0.05λ, resulting in point B. Point B is then inverted, yielding point C. $X_p = 258 \tan 18°$ ohms, and the normalized value of X_p is X_p/Z_o or simply $\tan 18° = +j0.325$. The inversion of X_p/Z_o is $Z_o/X_p = b_p/Y_o = -j3.08$. The coordinates of point C on the Smith chart are $1.41 - j0.42$. Adding the value of b_p/Y_o yields $1.41 - j3.50$ (point D). Inverting point D, a normalized impedance of $0.099 + j0.245$ (point E) results. Since this impedance is normalized to 258 ohms, the actual impedance at the driving point of the gamma is $(0.099 \times 258) + (j0.245 \times 258) = 25.5 + j63$ ohms. Minimum s.w.r. is obtained by connecting a capacitor of 63 ohms reactance between the coax cable and the gamma rod, but the minimum s.w.r. is 2.04 to 1 when using RG-8/U!

Now, if instead of making the driven element resonant it is shortened, the impedance Z_2 will consist of the normalized resistance of 0.6 ohm in series with a capacitive reactance. Shortening the typical 20-meter beam driven element by 12 inches will result in a reactance change of approximately 20 to 25 ohms. The normalized tap-point impedance thus will be about $0.6 - j0.6$ (point 1, Fig. 5B). The transferred radiation impedance to the input of the gamma (point 2) is $0.48 - j0.32$. Inverting this impedance gives the admittance (point 3) of $1.44 + j0.96$. Adding the value of b_p ($-j3.08$) the total normalized admittance becomes $1.44 - j2.12$ (point 4). Inversion of point 4 yields point 5, $0.219 + j0.322$. The impedance Z_T provided by the gamma match is then $(0.219 \times 258) + (j0.322 \times 258) = 56.5 + j83$ ohms. At 14 MHz. with a capacitor of 136 pf. in series with the gamma rod an s.w.r. of 56.5/52 = 1.08 is obtained. By changing the length of the gamma rod slightly a perfect match can be obtained.

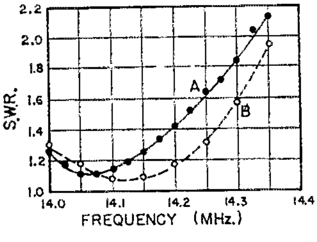
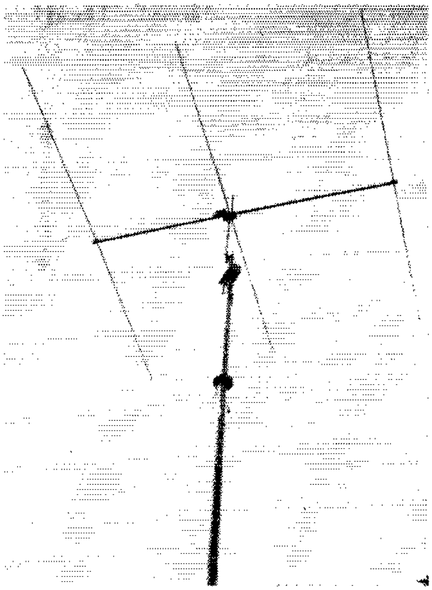


Fig. 6—Measured s.w.r. of W3PG three-element Yagi-Uda adjusted as described in the text, with gamma capacitance adjusted to minimize s.w.r. at 14.225 MHz. A—Based on impedance measurements with General Radio 916A r.f. bridge; B—Indicated by Jones Micromatch.



The three-element beam at W3PG. Driven-element length and gamma were adjusted according to the procedures developed in this article.

As noted in connection with Mushiake's analysis, changing the spacing between driven element and director, as well as changing director length, radically affects the reactance so that the driven-element physical length needed to make the impedance Z_2 exhibit the necessary capacitive reactance may vary considerably with different beam constructors, as indicated by Nose.[7]

Results

Based on the foregoing analysis of the gamma match, the driven element of the W3PG 14-MHz. Yagi was shortened to 32 feet 11 inches. Dimensions of reflector and director are 35 feet 9½ inches and 31 feet 7 inches, respectively, with approximately 0.204λ spacing to director and 0.185λ to reflector. Experiments were then made using a calibrated variable capacitor and antennascope to obtain matching resistance and capacitance as a function of the position of the gamma short. Good agreement was obtained using these results and working backwards on a Smith chart to obtain the radiation impedance. A radiation resistance of 20.8 ohms was obtained as an average value from the measurements. Length of the gamma rod is 56 inches, with the short 39 inches from the center of the driven element.

It was observed that the proximity of metal scaffolding used to form a work platform at the top of the pole affected the input impedance. The primary effect was modification of the reactive component of Z_2. For this reason a 150-pf. Johnson 2000-volt air variable was installed in a weatherproof box, coupled to a size 5 Selsyn follower so that the proper re-

(Continued on page 57)

are rated at ¼ ampere for a resistive d.c. load, ½ ampere peak. While this average current is not exceeded in most modern transmitters or transceivers using grid-block keying, key-click filters or r.f. bypass networks can cause much larger peak currents upon initial relay closure. When in doubt about the effective resistance in the keyed circuit of a grid-blocked transmitter, M & M recommends that an external resistor be placed in series with the relay contacts, its value being two ohms for each volt of grid-block bias. Similar precautions are applicable to cathode-keyed transmitters. Too much contact current may cause relay sticking and will shorten the life of the relay.

The self-contained sidetone generator and speaker is a convenient feature if the unit is used for code practice sessions. However, if the operator prefers headphones for copying received signals, it is necessary to remove the headphones to hear the sidetone during transmission, unless other means are available for monitoring one's "fist." No way is provided to disable the sidetone generator during operation.

Fig. 2 shows the schematic of the sidetone generator circuit. The oscillator is keyed by the driver transistor stage shown in block form. The driver stage acts as a switch to open and close the oscillator chassis return. No volume or pitch controls are provided. The pitch of the tone is perhaps higher than most c.w. operators prefer. In two units checked, the sidetone frequencies were measured as 1884 Hz. and 2143 Hz. In the second unit with the higher pitch, the manufacturer used a 0.22-μf. capacitor in place of the 0.33-μf. value shown in the schematic. Adding a 0.1-μf. capacitor in parallel with the 0.22-μf. value lowered the tone to 1514 Hz. A 0.22-μf. capacitor across the existing capacitor lowered the tone to 1047 Hz. Still larger values lowered the frequency even more, but sluggish sidetone generation resulted. Any of the frequencies obtained had a pleasing quality, and the higher pitches were not objectionable to this writer.

Only one difficulty was experienced with the "DAH-DITTER" during on-the-air tests. The keying wire externally connected to the dash screw terminal became intermittently shorted to the head of the adjacent screw extending through the front panel for securing the printed circuit board. This mounting screw is grounded at the board. The sympton of the problem was intermittent and seemingly delayed dash generation. Of course the cure was simply to dress the lead properly.

The EK-1 keyer is a compact and well-constructed unit. An instruction manual with detailed information on connection to the transmitter and on operation is included. The instructions also show how an ordinary "bug" keyer may be used with the EK-1. As a trouble-shooting aid, the schematic includes voltage measurements taken throughout the circuit, although removal of the integrated circuits from the printed circuit board without special tools is not recommended. As a rule, integrated circuits operate for thousands of hours without failure, so a defective IC is unlikely, but factory service is available at a reasonable fee if required.
— *K1PLP*

M & M Electronics EK-1 Dah-Ditter Electronic Keyer

Height: 6¼ inches.
Width: 3¾ inches.
Depth: 2¼ inches.
Weight: 26½ ounces.
Power Requirements: 110 volts, 50- to 400-Hz. a.c., 5 watts.
Price Class: $35.
Manufacturer: M & M Electronics, 6835 Sunnybrook, N.E., Atlanta, Georgia 30328.

The Gamma Match

(Continued from page 15)

actance could be inserted by adjustment of a size 5 Selsyn generator at the transmitter when the beam was in final position about seven feet above the top of the pole.

When s.w.r. measurements were made at the transmitter it was found that a perfect match could be obtained over the c.w. portion of 14 MHz. by adjustment of the capacitor at each frequency, but with the 39-inch rod length and the other beam dimensions used, minimum s.w.r. at 14.25 MHz. occurred with the maximum capacitance of 150 pf. Fig. 6 shows the s.w.r. *vs.* frequency when maximum capacitance is used.

Conclusions

As a result of the trouble experienced with the initial installation of a wide-spaced 14-MHz. Yagi-Uda that employed different element spacing and lengths than previously used at W3HEC, an analysis of the matching problem has been made that seems to be valid on the basis of the experimental results obtained. It was found that matching required a shortened driven element when the Z_o of the transmission line formed by the gamma rod and driven element is on the order of 250 to 300 ohms. Further computation was made for $Z_o = 550$ to 600 ohms and it was found, using the analysis described, that a resonant driven element could be matched using the higher Z_o. However, bandwidth may not necessarily be improved using the resonant driven element and higher-Z_o matching section. As seen from Fig. 6, the bandwidth when tuned for a match at 14.225 MHz. is adequate without readjusting the gamma section for operation in the c.w. portion of the band. (The s.w.r. was obtained using a G-R 916A bridge, and impedance measurements obtained with the bridge were plotted on a Smith chart to obtain s.w.r.) Performance of the beam is good, a front-to-back ratio of 20 db. is obtained, and the pattern width appears to be as expected.